Improved Performance-complexity Trade-offs for Soft-decision Decoding by Channel-polarized Multilevel Coding

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Abstract Channel-polarized multilevel coding (CP-MLC) improves the net coding gain of the concatenated codes by 0.07 to 0.20 dB under short-block soft-decision FEC and KP4 codes while simultaneously reducing the complexity by 37.5% or more. ©2023 The Author(s)

Introduction
With the advent of artificial intelligence, 5G, and cloud data services, the internet traffic of data center (DC) applications is increasing. To build economical networks, a significant number of low-cost and low-power optical transceivers are required [1]. Forward error correction (FEC) is one of the most important technologies to construct highly reliable communication systems. Due to implementation penalties, performance optimization under strict power constraints is essential, as small signal-to-noise ratio (SNR) improvements result in significant optical SNR improvements in DC applications [2]. To achieve high decoding performance and low-power consumption, concatenated low-complexity soft-decision (SD) and hard-decision (HD) FEC codes have been studied [3–5]. In next-generation Ethernet, a configuration has been discussed where the modern switch chip has the outer KP4 code and the inner SD-FEC code is applied to the optical transceiver module [6], as shown in Figure 1. The inner low-complexity SD decoding, such as Chase-2 decoding [7], significantly increases the decoding complexity with longer block lengths, and the theoretical performance limit degrades as the block length decreases [8–9].

In this study, we develop low-complexity FEC codes with a FEC overhead (FEC-OH) of 12 to 13% using channel-polarized multilevel coding (CP-MLC), which reduces the decoding complexity while mitigating the performance degradation [10], for DC applications. We show that CP-MLC using short-block-length eBCH and KP4 codes outperforms the near maximum likelihood decoding (MLD) performance of concatenated eBCH-KP4 codes by optimizing the inner-code error floor under KP4 bit-error-rate (BER) threshold. Our simulation results also demonstrate that eBCH-KP4 CP-MLC improves the performance-complexity trade-offs compared to concatenated eBCH-KP4 codes.

Channel-polarized multilevel coding for DC applications
Figure 1 shows the configuration for applying CP-MLC to DC applications. To effectively reduce SD-FEC, CP-MLC uses encoder-side exclusive-or (XOR) and two-stage decoding to transform the bit reliability and applies SD-FEC only to the unreliable bits. The outer KP4 code on the switch is the same as the concatenated code. The special cases of the CP-MLC using eBCH and KP4 codes are proposed in the 800LR [11–12]. On the encoder side, CP-MLC encodes information bits $b$ to the outer codeword $z = (z^{(1)}, z^{(2)}, z^{(3)}, ..., z^{(d)})$, where $d$ is the number of lanes. The inner encoder transforms $z^{(1)}$ to $z^{(1)}$ in only the top lane. The CP-block performs XOR, outputting $x^{(1)} = z^{(1)} ⊕ z^{(2)} ⊕ ... ⊕ z^{(d)}$ and bypassing $x^{(i)} := z^{(i)}$ in other lanes. On the decoder side, the log-likelihood ratio (LLR) $l = (l^{(1)}, l^{(2)}, ..., l^{(d)})$ is updated to

$$\lambda = SD(l) := l^{(1)} ⊕ l^{(2)} ⊕ ... ⊕ l^{(d)},$$

where $a ⊕ b := 2 \tanh^{-1}(\tanh(a/2) \tanh(b/2)) \approx \text{sign}(a)\text{sign}(b)\min(|a|,|b|)$ and $\text{sign}(b) := b/|b|$ [13]. The SD-FEC decoder recovers the codeword $\hat{z}^{(1)}$ and obtains the estimated

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Fig.1. Schematic diagram of transmitter (Tx) and receiver (Rx) with BCH-KP4 CP-MLC and concatenated BCH-KP4 codes.
information \( \hat{z}^{(1)} \) from \( \lambda \). The HD-block calculates

\[
z^{(i)} = HD^{(i)}(\hat{z}^{(i)}) = \frac{1 - \text{sign}\left(\phi^{(i)}(\hat{z}^{(i)})\right)}{2}, \tag{2}
\]

where

\[
\phi^{(i)}(\hat{z}^{(i)}) = t^{(i)} + (-1)^{i}t^{(1)}t^{(2)} \otimes \cdots \otimes t^{(i-1)}t^{(i+1)} \otimes \cdots \otimes t^{(d)}
\]

is defined [13].

The outer decoder recovers the information from \( \hat{z} \). The decoding rate is given by

\[
R_{CP} = R_{HD}(d-1 + R_{SD})/d,
\]

where \( R_{SD} \) and \( R_{HD} \) are the coding rate of SD- and HD-FEC codes, respectively.

**Complexity comparison of concatenated codes and CP-MLC with Chase-2 decoding**

The Chase-2 decoding first performs sorting based on the magnitude of the LLR and generates the test patterns (TPs) by flipping the least reliable bits. Next, the all-candidate codewords are calculated from each TP, by using the HD decoding (HDD). Finally, the Chase-2 decoding outputs the estimated codeword, which is a candidate codeword with a minimum Euclidean distance from the received value. The searching for the candidate codeword has a relatively high complexity compared to sorting and selecting because the HDD is executed on all TPs. Thus, the complexity of Chase-2 decoding is evaluated by the number of TPs [14–15]. CP-MLC can reduce the proportion of Chase-2 decoding in the entire throughput to \( 1/d \), but extra processing steps of Eqs. (1) and (2), are required. However, the complexity of Eqs. (1) and (2) are significantly smaller than that of the Chase-2 decoding, because they consist of a few min and sign functions. Thus, we evaluate the decoding complexity by using the number of TPs/\( d \) (denoted as TP/d) under the same throughput.

Note that the complexity of concatenated codes is represented by TP/1 because it can be regarded as CP-MLC with \( d = 1 \).

**Simulation results**

We constructed concatenated eBCH-KP4 codes with FEC-OH of 12.9% and CP-MLC with FEC-OH of 12.2 to 12.4% for each \( d \). The KP4 code has a coding rate \( R_{KP4} = 514/544 \) and a BER threshold of \( 2.2 \times 10^{-4} \) required to achieve the post-FEC BER equal to \( 10^{-15} \) [16]. \( (n,k,2t+2) \)-eBCH codes, characterized by the block length as \( n \), information length as \( k \), and minimum distance as \( 2t + 2 \), can correct \( t \)-bit errors and detect \( (t + 1) \)-bit errors. Our concatenated eBCH-KP4 code has \((128,120,4)\)-eBCH codes. In comparison, our eBCH-KP4 CP-MLC with \( d = 2, 3, \) and 4 have \((128,113,6), (128,106,8), \) and \((128,99,10)\)-eBCH codes, respectively. Note that CP-MLC has high FEC-OH for eBCH codes due to bypassed bits. eBCH decoder has the Chase-2 decoding, where HDD of the processing step of candidate search adopts the Berlekamp-Massey algorithm. Figure 2 shows the post-inner-FEC BER of concatenated codes and CP-MLC for each TP of the Chase-2 decoding, under binary phase-shift keying (BPSK) and additive white Gaussian noise (AWGN) channels. Note that the post-inner-FEC BER of CP-MLC includes bypassed reliable bits. CP-MLC has large BER on the low \( E_b/N_0 \) region compared to concatenated codes because of
with unreliable bits, decoding with TP = 1024 for each minimum distance d of BCH codes and for concatenated eBCH codes, entire bits, unreliable bits, and reliable bits for eBCH-KP4 CP-MLC with d = 2 and TP = 1024. Fig. 3. (a) Block error rate (not BER) vs. Eb/N0 of Chase-2 decoding with TP = 1024 for each minimum distance d of BCH codes and (b) post-inner-FEC BER vs. Eb/N0 for concatenated eBCH codes, entire bits, unreliable bits, and reliable bits for eBCH-KP4 CP-MLC with d = 2 and TP = 1024.

The decoding performance of concatenated codes and CP-MLC with d = 2 saturated rapidly as shown in Fig. 2(a) and (b), because the minimum distance of BCH codes was small and many candidate codewords were found with few TPs. CP-MLC has an error floor caused by bit error of bypassed reliable bits Fig. 2(c) and (d). Thus, the decoding performance is constricted for large d.

Next, we show through numerical simulation analysis that CP-MLC outperforms concatenated eBCH-KP4 codes under Chase-2 decoding with TP = 1024. Although strict achieving MLD by using Chase-2 decoding requires HDDs with the number of times by $2^N$, for BCH codes with a small minimum distance d, the Chase-2 decoding with TP = 1024 shows a performance close enough to MLD, as shown in Fig. 3(a). Note that we used non-extended $(127, 127-t, t)$-BCH codes to compare with the result of MLD [17]. We evaluated the post-inner-FEC BER when using Chase-2 decoding with TP = 1024 and BER of unreliable/reliable bits of CP-MLC, as shown in Fig. 3(b). Unlike the post-inner-FEC BER of the entire bit, note that we assumed $z^{(1)} = \hat{z}^{(1)}$ in the BER of bypassed-reliable bits to accurately estimate the error floor except for the effect of error propagation. The decoding performance improves in moderate BER regions the error floor is higher due to bypassed-reliable bits. CP-MLC achieved up to a 0.20 dB improvement compared with the concatenated code with near-MLD performance, at the KP4 BER threshold as shown in Fig. 3(b).

Finally, we evaluate the net coding gain (NCG), which is defined by

$$ NCG = RSNR_{PSK} - RSNR_{inner} - 10log_{10} R_{KP4}, $$

where $RSNR_{PSK}$ means the required SNR (RSN) of uncoded BPSK to achieve the BER of $10^{-5}$. The RSN of inner codes $RSNR_{inner}$ is obtained from the cross point of the KP4 BER threshold and post-inner-FEC BER. Figure 4(a) shows the NCG for each TP. NCG envelope of CP-MLC outperforms NCG for concatenated codes, while CP-MLC with d = 4 cannot improve concatenated codes at large TPs because it suffers the error floor. As shown in Fig. 4(b) and (c), CP-MLC reduced the decoding complexity by about 37.5% or more and improved performance by about 0.07 to 0.20 dB at the same TPs, and reduced the number of TPs at the same NCG, respectively.

**Conclusion**

We demonstrated that eBCH-KP4 CP-MLC can improve the decoding performance of near MLD by about 0.20 dB, providing better decoding performance-complexity trade-offs compared to concatenated eBCH-KP4 codes. As a result, CP-MLC is able to reduce decoding complexity by approximately 37.5% or more, while simultaneously improving performance by approximately 0.07 to 0.20 dB.
References


